ATOMIC ENERGY CENTRAL SCHOOL # 4, RAWATBHATA

MODEL PAPER FOR HALF YEARLY EXAMINATION, 2015

CLASS XII

TIME: 3 hrs SUBJECT – MATHEMATICS MM: 100

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 26 questions divided into three sections A, B, C. Section A comprises of 6 questions of one marks each. Section B comprises of 13 questions of four marks each and section C comprises of 7 questions of six marks each.

SECTION A

1. If $f(x) = 8x^3$ and $g(x) = x^{1/3}$, find gof and fog.

2. .Prove that $\frac{9\pi}{8} - \frac{9}{4} Sin^{-1}\frac{1}{3} = \frac{9}{4} Sin^{-1}\frac{2\sqrt{2}}{3}$

3. Find x if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

4. Find the value of the determinant $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix}$

5. Evaluate $\int \frac{(Cosx - Sinx)}{1 + Sin2x} dx$.

6. Evaluate $\int e^x Secx(1 + Tanx)dx$.

SECTION B

7. If R_1 and R_2 are equivalence relations in a set A, Show that $R_1 \cap R_2$ is also equivalence relation.

OR

Let A and B be sets. Show that f : A×B → B×A such that f(a, b) = (b, a) is bijective.

8. Prove that $Tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2} Cos^{-1}x$, $-\frac{1}{\sqrt{2}} \le x \le 1$.

9. Evaluate $Tan\left[2Tan^{-1}\left(\frac{1}{5}\right) + \frac{\pi}{4}\right]$

10. Prove that

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1 - x^3)^2$$

11 Two schools A and B decided to award prizes to their students for three values honesty (x), punctuality (y) and obedience (z). School A decided to award a total of Rs. 11000 for the three values to 5, 4 and 3 students respectively while school B

decided to award Rs. 10700 for the three values to 4, 3 and 5 students respectively. If all the three prizes together amount to Rs. 2700, then.

i. Represent the above situation by a matrix equation and form Linear equations using matrix multiplication.

ii. Is it possible to solve the system of equations so obtained using matrices?

iii. Which value you prefer to be rewarded most and why?

12. Let A = $\begin{bmatrix} 0 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 0 \end{bmatrix}$. Show that (I + A) = (I – A).$\begin{bmatrix} Cos\theta & -Sin\theta \\ Sin\theta & Cos\theta \end{bmatrix}$

13. Let f(x) = $\begin{cases} \frac{b(1-Sinx)}{(\pi-2x)^2}, & if\ x > \frac{\pi}{2} \\ a & if\ x = \frac{\pi}{2} \\ \frac{1-Sin^3 x}{3Cos^2 x} & if\ x < \frac{\pi}{2} \end{cases}$. If f(x) be a continuous function at x=$\frac{\pi}{2}$, find a and b.

14. If y = $(x)^{Cosx} + (Sinx)^{Tanx}$, find $\frac{dy}{dx}$

OR

If x = a (Cos θ + θ Sin θ) and y = a (Sin θ - θ Cos θ), find $\frac{dy}{dx}$

15. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

16. Sand is pouring from a pipe at the rate of 12 cm^3/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

OR

Using differentials, find the approximate value of $\sqrt{0.037}$

17. Evaluate $\int \frac{\sqrt{x^2+1}\,[Log(x^2+1)-2Logx]}{x^4}dx$.

OR

Evaluate $\int \frac{x+2}{\sqrt{(x-2)(x-3)}}dx$

18. Evaluate $\int \frac{Sinx+Cosx}{9+16Sin\,2x}dx$

19. Evaluate $\int_0^{\pi} \frac{x}{a^2Cos^2x+b^2Sin^2x}$dx

SECTION C

20. Let $f : \mathbf{N} \rightarrow \mathbf{R}$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : \mathbf{N} \rightarrow S$, where, S is the range of f, is invertible. Find the inverse of f.

21. If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations

$2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$.

22. If $(x - a)^2 + (y - b)^2 = c^2$, for some $c > 0$, prove that

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$ is a constant independent of a and b.

23. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

24. Find the intervals in which the function f given by $f(x) = \frac{4Sinx - 2x - xCosx}{2 + Cosx}$ is increasing or decreasing.

OR

For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

25. Prove that $\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz\,(x + y + z)^3$

26. Evaluate $\int \sqrt{Cotx}\, dx$

OR

Evaluate $\int \frac{x^2 + x + 1}{(x+1)^2 (x+2)}\, dx$
